JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 10, No. 4, October–December 1996

Gravitational Effects on Laminar Convection to Near-Critical Water in a Vertical Tube

S. H. Lee* and J. R. Howell†
University of Texas at Austin, Austin, Texas 78712-1063

Numerical calculation is performed to study the strongly coupled heat transfer and flow of water at near critical conditions in laminar flow in a vertical tube. Near the critical region fluid flow and heat transfer depend on the temperature and pressure because of large property variations that include transition between liquid-like behavior and gas-like behavior. The two-dimensional (r, z) model includes the effects of variable thermodynamic and transport properties such as density, specific heat, viscosity, and conductivity. Heat transfer coefficient and profiles of temperature and velocity are shown in the developing region of the tube. The very large influence of gravity on the two-dimensional flow and temperature profiles in the tube is shown. The effect of proximity to the pseudocritical point is considered. The tube wall is heated with constant wall heat flux and uniform flow is assumed in the inlet of the tube. Inlet fluid temperature, pressure, and Reynolds number are used as the main parameters.

Nomenclature

C_p = specific heat at constant pressure, J/kg
D = tube diameter, m
f = friction factor
Gr = Grashof number, $\rho_{in}^2 g \beta_{in} Q_w R^4 / k_{in} \mu_{in}^2$
g = acceleration caused by gravity, m/s^2
h = convective heat transfer coefficient, W/m^2 K
k = thermal conductivity, W/m K
L = tube length, m
$\dot{m}$ = mass flow rate, kg/s
P_R = reduced pressure, p/p_c
Pr = Prandtl number
p = pressure, N/m^2
Q = heat flux, W/m^2
R = radius, m
Re = Reynolds number at inlet, $\rho_{in} u_{in} D / \mu_{in}$
r = radial distance, m
T = temperature, K
T_{pc} = pseudocritical temperature, K
u = local velocity in axial direction, m/s
v = local velocity in radial direction, m/s
y = nondimensional distance from the wall, $1 - (r/R)$
z = axial distance, m
α = nonuniform parameter
β = compressibility, 1/K
ΔT = wall to bulk temperature difference, K
θ = dimensionless temperature, $k_{in}(T - T_{in})/Q_w D$
μ = absolute viscosity, kg/s m
ρ = density, kg/m^3
τ = shear stress, N/m^2
ϕ = function variable

Subscripts
b = bulk condition
c = critical point value
in = inlet condition
w = wall condition

Superscripts
i = iteration counter

Introduction

EXTENSIVE investigations have been carried out theoretically and experimentally on mixed convection problems in a vertical tube, with increased interest recently on fluids near the critical region. The heat transfer coefficient in this region shows different characteristics from the constant-property-based solution caused by wide variations in fluid properties. Practically, the supercritical condition is important in the design of heat exchange equipment that uses water in powerplants, hydrogen as a working fluid for rockets, helium as a coolant for superconducting equipment, and carbon dioxide in supercritical extraction systems, etc.

Near the critical region, there are large variations of thermodynamic and transport properties shown for water in Fig. 1. As the fluid temperature approaches the pseudocritical temperature at a specified pressure, the variation of fluid properties becomes larger. The properties change from those of a liquid-like phase to those of a gas-like phase as the temperature increases across the pseudocritical temperature. Hence, in treating buoyancy effects, the Boussinesq approximation that assumes linear dependence of density on temperature may not be reasonable because the thermal expansion coefficient also has large variation along with other properties. The degree of variation depends on both fluid temperature and pressure.

Most experimental[1–4] and theoretical[5–8] investigations of fluid flow and heat transfer near the critical region are for turbulent flow because of the need for the design of practical systems. These references report on the degradation phenomenon of the heat transfer coefficient in a vertical tube for high flux. For laminar flow in a vertical tube near the critical region less research has been done. Koppel and Smith[9] solved numerically the case of laminar upward flow in a vertical tube for supercritical CO_2. They modeled the case of constant wall heat flux using implicit finite difference equations neglecting the radial velocity component. Their results indicated differences between near-critical and constant property results for the heat transfer coefficient, especially in the developing region near the pseudocritical point. They didn't consider the possibility of recirculating flow. Dashevsky et al.[10,11] also solved numerically the case of laminar upward flow in a tube for near-critical helium and CO_2. The boundary-layer approximations are used and the velocity profile is determined using the Boussinesq approximation. They formulated finite difference

Received Oct. 30, 1995; revision received April 15, 1996; accepted for publication April 16, 1996. Copyright © 1996 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.
*Graduate Student, Department of Mechanical Engineering.
†Baker-Hughes Centennial Professor, Department of Mechanical Engineering. Fellow AIAA.

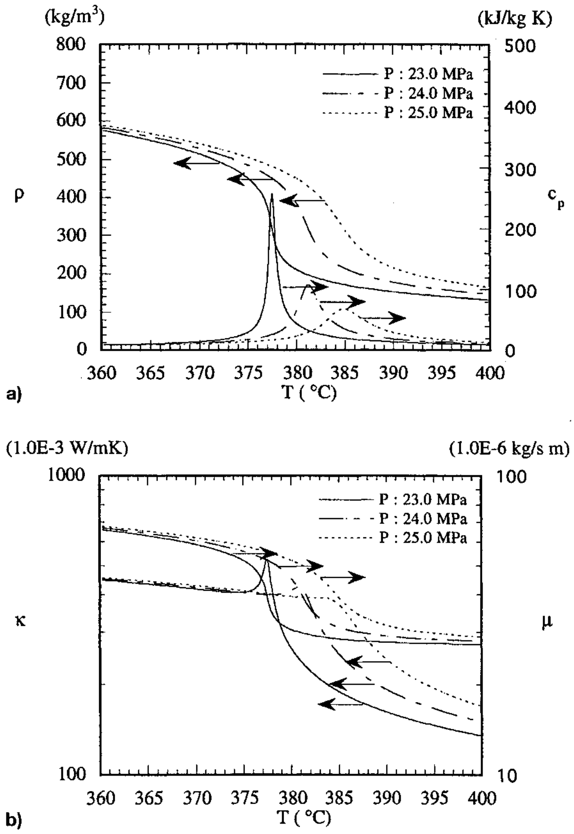

Fig. 1 Thermodynamic and transport properties variations for water near the critical region: a) density and specific heat and b) viscosity and thermal conductivity.

schemes for the general case including recirculating flow. Their calculations are in good accord with experimental results. Bogachev et al.[12] and Vlakhov et al.[13] suggested experimental correlations for the heat transfer coefficient for supercritical fluids in the laminar/transitional flow region. Their results show that the heat transfer coefficient has a maximum value in the developing region of the tube, and far from the critical region the heat transfer characteristics are similar to those for the constant property case.

These experimental and numerical results show that there are severe variable property effects on fluid flow and heat transfer near the critical region. Despite several studies of vertical tube laminar flow for near critical fluids, the effect of various approximations on computation of heat transfer and fluid flow are not completely understood. The purpose of this study is to investigate the basic mechanism of fluid flow and heat transfer and for predicting laminar-upward flow convection heat transfer in a vertical tube. Water is used as the fluid near the critical region, and this work does not rely on the boundary layer or Boussinesq approximations.

Numerical Modeling

The problem to be analyzed is heat transfer to near-critical water flowing through a vertical smooth-walled circular tube. Flow enters the tube with a uniform velocity and temperature. The flow is assumed to be steady state and axisymmetric, and in thermodynamic equilibrium. The governing continuity equation, Navier–Stokes equations, and energy equation in axisymmetric coordinates, which are used in this modeling are shown in the following. Wall thermal conductivity is assumed so high that there is no temperature gradient across the wall.

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v) + \frac{\partial}{\partial z}(\rho u) = 0 \tag{1}$$

Momentum:

$$\rho v \frac{\partial v}{\partial r} + \rho u \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) - \frac{\tau_{\theta\theta}}{r} + \frac{\partial}{\partial z}(\tau_{rz}) \tag{2}$$

$$\rho v \frac{\partial u}{\partial r} + \rho u \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial z} - \rho g + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial}{\partial z}(\tau_{zz}) \tag{3}$$

where

$$\tau_{rr} = \mu\left[2\frac{\partial v}{\partial r} - \frac{2}{3}(\nabla \cdot V)\right]$$

$$\tau_{\theta\theta} = \mu\left[2\frac{v}{r} - \frac{2}{3}(\nabla \cdot V)\right]$$

$$\tau_{zz} = \mu\left[2\frac{\partial u}{\partial z} - \frac{2}{3}(\nabla \cdot V)\right]$$

$$\tau_{rz} = \mu\left[\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right]$$

Energy:

$$\rho C_p \frac{\mathrm{d}T}{\mathrm{d}t} = \mathrm{Div}(k\nabla T) + \beta T\frac{\mathrm{d}p}{\mathrm{d}t} + \mu\phi \tag{4}$$

The thermodynamic and transport properties are taken to be functions of temperature and pressure. For thermodynamic and transport properties of water, the computer code of Lester et al.[14] is used. The associated boundary conditions of the model are

$z = 0$ (inlet) $\quad u = u_{\mathrm{in}},\ v = 0,\ T = T_{\mathrm{in}},\ P = P_{\mathrm{in}}$

$0 < z < L \quad$ at $r = 0$: $\quad \dfrac{\partial u}{\partial r} = 0,\ \dfrac{\partial v}{\partial r} = 0,\ \dfrac{\partial T}{\partial r} = 0$

at $r = R$: $\quad u = 0,\ v = 0,\ Q_w = -k\dfrac{\partial T}{\partial r}$

At the outlet, which is placed 10 diameters downstream of the tube inlet, linear extrapolation boundary conditions are used for u and T. Results are presented for only the single tube length, but greater lengths showed little change in the results for $z \leq 10D$.

The governing equations are solved with the SIMPLE algorithm[15] using the power-law scheme. A staggered grid system is used and relaxation factors are used for stability. The convergence is checked by computing the normalized mass residual in the equation of continuity. The iteration is stopped when the residual is less than 10^{-3}. Successive iteration values of each variable should also satisfy the following convergence criteria:

$$\left|\frac{\phi^{i+1} - \phi^{i}}{\phi^{i}}\right|_{\max} < 10^{-3}$$

where ϕ: u, v, and T.

For numerical calculation, orthogonal grids that are distributed uniformly along the axial direction and nonuniformly in the radial direction (clustered near the wall) are used. The following relation is used in the radial grid system:

$$y = \frac{(\alpha + 1) - (\alpha - 1)\{[(\alpha + 1)/(\alpha - 1)]^{1-\bar{y}}\}}{[(\alpha + 1)/(\alpha - 1)]^{1-\bar{y}} + 1}$$

where $\bar{y}$ is the radially uniform grid position and nonuniform parameter, and α is 1.05.

To check the grid size effect on calculation results, axial grids of 50, 100, and 200 points and radial grids of 20, 40, and 80 were used. The difference between the calculation results of heat transfer coefficient in grid points of 100 × 40 and 200 × 80 is less than 1%. Computation is carried out on at least the 100 × 40 grid. The validity of the extrapolation boundary condition at the exit of the tube was checked by comparison with the results for longer tubes. To further verify this program code, other results[16,17] of developing flow in the tube were compared. The predictions agree very well. All calculations are carried out on an IBM RISC System/6000.

Results and Discussion

For heat transfer near the critical region, the fluid may pass through the pseudocritical temperature at which there is the maximum property gradient with temperature. These property variations are related to the transition between liquid-like and gas-like behavior across the pseudocritical point, and these effects are interrelated with the flow and temperature fields. Especially in upward flow with heating, the buoyancy effect caused by density variation is important, as shown in the following results.

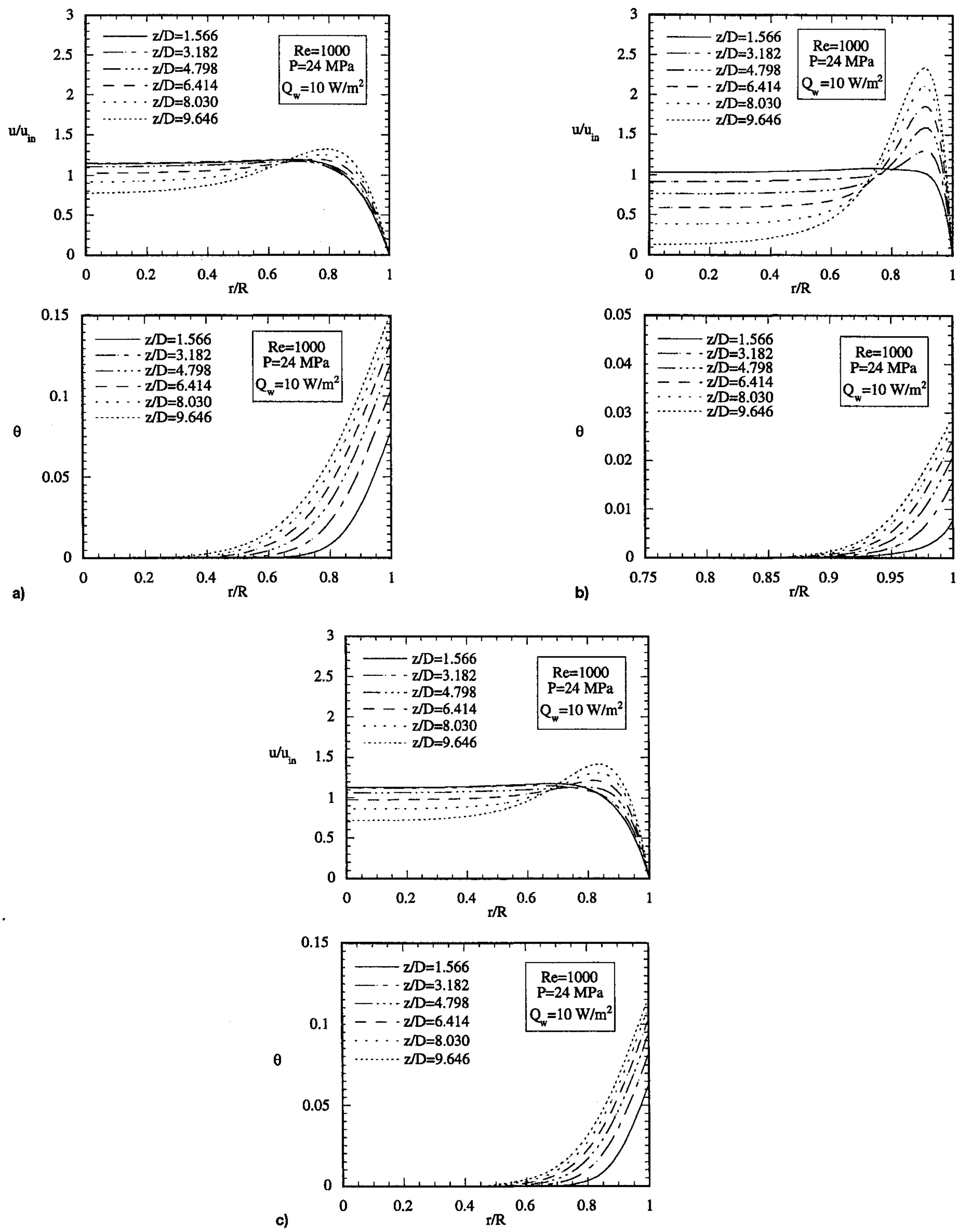

Fig. 2 Comparison of velocity and temperature distributions along the tube with fluid inlet temperature. T_{in} = a) 300, b) 375, and c) 450°C.

The Prandtl, Reynolds, and Grashof numbers are the usual parameters for the velocity profile and heat transfer in tube flow with mixed convection. However, near the critical region these nondimensional numbers are not constant and their variations are severe. There is a pseudocritical temperature where there is maximum variation of properties for each pressure. In this analysis, the temperature, pressure, and Reynolds number of the inlet fluid are used as the main parameters. Additional nondimensional numbers are calculated based on the inlet condition.

Figure 2 shows velocity and temperature profiles along the tube in the entrance region in the vertical tube for the case of a very low constant wall heat flux of 10 W/m^2. The velocity profiles show that the buoyancy effect caused by density variation near the wall is large compared with the shear stress-induced deceleration of the fluid. The velocity of the fluid near the wall increases along the tube from the inlet and the velocity peak near the wall will continue to increase until buoyancy effects become negligible. It is also seen that the centerline velocity continues to decrease, which means that recirculation may occur in the downstream core region of the tube. Such flow reversal was seen for results at higher wall heat flux. The centerline velocity decrease rate depends on many parameters. For the near-critical region, the inlet condition is important because the variation of properties is different for each pressure and temperature.

The radial velocity component has a direction toward the wall over much of the tube radius to satisfy mass conservation, which means that streamlines move toward the wall along the tube. The temperature profiles show that a very narrow region near the wall has variations of temperature, and in this region there is a resulting variation of properties such as density, specific heat, viscosity, and conductivity. Note that the temperature distribution is not as severely deformed as might be expected with these property variations.

Temperature profiles in the tube are also affected by the buoyancy force. As the inlet fluid temperature approaches the pseudocritical temperature, buoyancy effects on the forced convection increase rapidly, resulting in increased fluid velocity near the heated wall, and decreased thermal boundary-layer thickness. The pseudocritical temperature at 24.0 MPa is about 382°C.

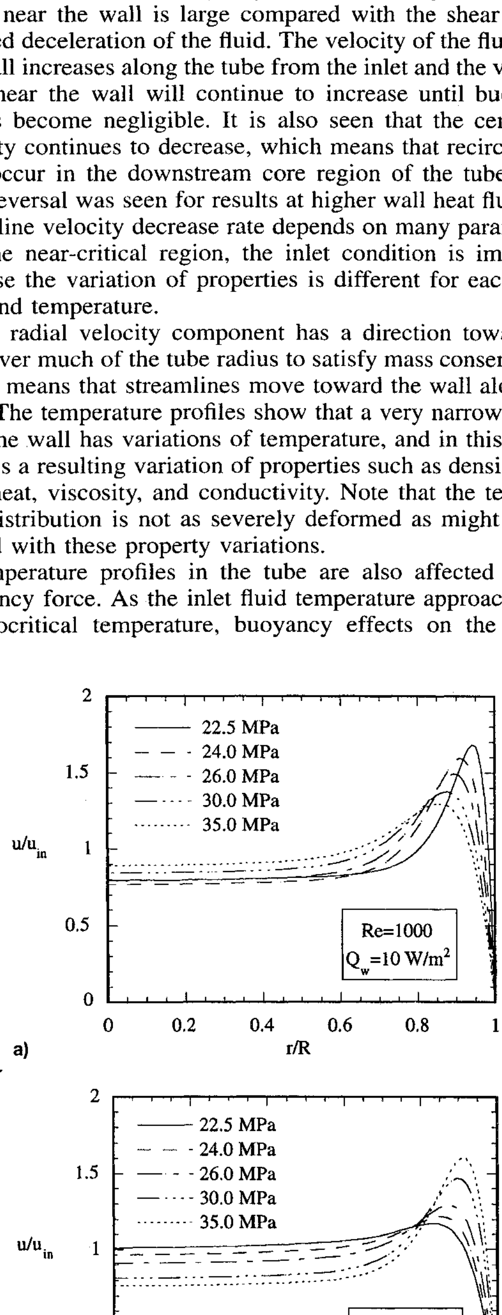

Fig. 3 Comparison of velocity distributions with pressure at a position of z/D = 4.8. T_{in} = a) 375 and b) 425°C.

As the fluid temperature approaches the pseudocritical temperature, the density variation with temperature increases very rapidly and this variation also depends on the pressure. Hence, the buoyancy force varies with the fluid temperature for a specified pressure and the flowfield is strongly affected. Figure 3 shows the effects of inlet pressure on the velocity and temperature profiles. The pseudocritical pressure of 375°C is about 22.1 MPa. Then for the fluid temperature of 375°C, the buoyancy force increases as the pressure is decreased toward the critical pressure. As the pressure decreases, the velocity profile is more distorted and the thermal boundary-layer thickness decreases. However, for the fluid temperature of 425°C, the velocity profile is less distorted as the pressure decreases toward the critical point pressure. Thus, the buoyancy force dependence on pressure changes between these two fluid temperatures.

Figure 4 shows the buoyancy force variation with respect to fluid temperature at several pressures for a constant wall heat flux of 10 W/m^2 in a vertical tube. The effect of buoyancy force in a vertical tube may be represented by the parameter[18] of *Gr*/*Re*. As the fluid temperature approaches the pseudocritical temperature, *Gr*/*Re* becomes larger and reaches a peak value at the pseudocritical point. The maximum value of *Gr*/*Re* increases as the pressure approaches the critical point (T_c = 374°C, P_c = 22.05 MPa). The buoyancy force has steep change near the pseudocritical temperature, and as the pressure approaches P_c, it becomes even steeper. This is the chief reason for the distorted velocity profiles, even at low wall heat flux. From this figure it can be seen that *Gr*/*Re* at T_{in} = 375°C is about 100 times larger than at T_{in} = 500°C and T_{in} = 200°C.

The heat transfer coefficient can be defined as

$$h = Q_w/(T_w - T_b) \tag{5}$$

and T_b is defined as

$$T_b = \int_A \rho u C_p T \, \mathrm{d}A \bigg/ \int_A \rho u C_p \, \mathrm{d}A \tag{6}$$

Figure 5 shows the heat transfer coefficient distributions along the vertical tube for several Reynolds numbers. Unlike forced convection in a tube for a fluid with constant properties, as the mass flux (*Re*) increases the heat transfer coefficient decreases after some distance. This effect is related to the higher velocity peak near the wall for lower inlet mass flux. This means that in the entrance region the buoyancy effect is very important. The heat transfer coefficient including the gravitational body force has a relatively large value compared with the result neglecting the gravity force.[18] This difference

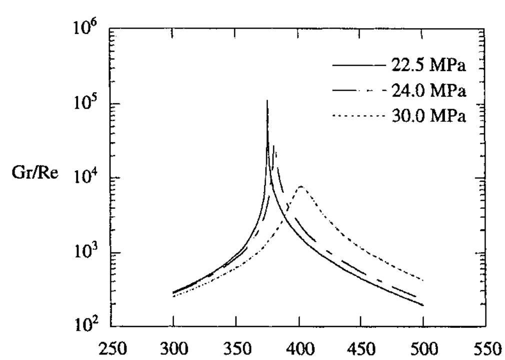

Fig. 4 Buoyancy force parameter dependence on water temperature and pressure; Q_w = 10 W/m^2, Re = 1000.

is caused by the high convection caused by rapid acceleration of the fluid near the wall because of the buoyancy force.

The heat transfer coefficient for various inlet fluid temperatures is shown in Fig. 6. The results show that the heat transfer coefficient rapidly decreases along the vertical tube but the variation with distance for $z/D > 5.0$ is relatively small. Also, the heat transfer coefficient increases as the inlet fluid temperatures approaches the pseudocritical temperature. This is related to the high buoyancy effect as shown in the velocity profile. When the inlet fluid temperature is near the pseudocritical temperature, free convection becomes more important than forced convection. High acceleration near the wall thins the thermal boundary layer, which results in large heat transfer coefficient. The pseudocritical temperature for 24 MPa is about 382°C; below this temperature the fluid has liquid-like behavior and above it gas-like behavior. Hence, with the same Reynolds number the heat transfer coefficients for $T_{in} > 382°C$ are lower than for $T_{in} < 382°C$.

Figure 7 shows the heat transfer coefficient for various pressure conditions, with trends similar to those for various inlet fluid temperatures. As the pressure decreases toward the critical pressure, the heat transfer coefficient increases. For pressures above 30 MPa, the difference in heat transfer coefficient is relatively small and is because of small changes of properties in the liquid phase.

The friction factor is defined as

$$f = 8\tau_w/\rho_{in}u_{in}^2 \tag{7}$$

and Fig. 8 shows the friction factor distribution along the tube. Because of the buoyancy, the fluid velocity near the wall increases gradually along the tube, and so for lower mass flow rate, the friction factor becomes larger. It can be seen that the friction factor also depends on inlet fluid temperature. As the fluid temperature approaches the pseudocritical temperature, the friction factor increases because of high buoyancy. Even with the decrease of viscosity at the wall along the tube from increasing fluid temperature, the increase of velocity gradient is so large that the wall shear stress increases, causing the friction factor to increase along the tube from near the entrance. These friction factors in the developing region of the tube are large compared with the fully developed laminar tube flow friction factor for constant properties, which is

$$f = 64/Re \tag{8}$$

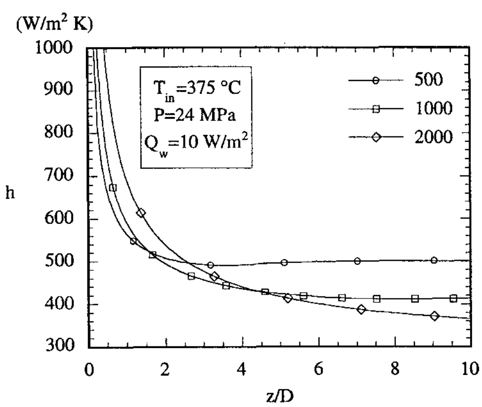

Fig. 5 Heat transfer coefficient distributions along the tube for various Reynolds numbers.

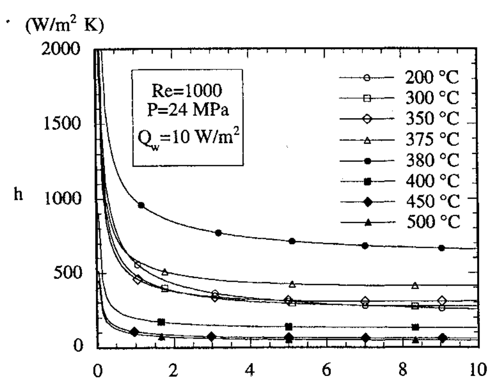

Fig. 6 Heat transfer coefficient distributions along the tube for various inlet water temperatures.

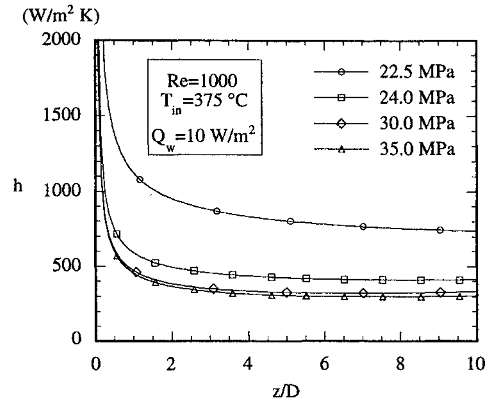

Fig. 7 Heat transfer coefficient distributions along the tube for various water pressures.

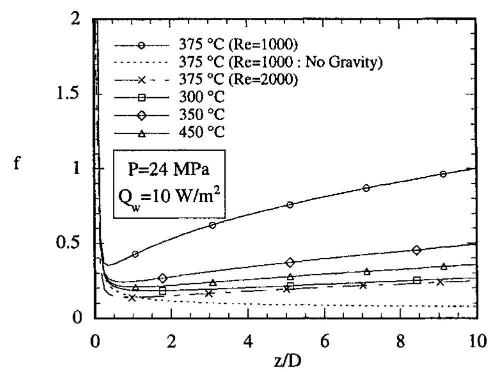

Fig. 8 Friction factor distributions along the tube for various Reynolds numbers and inlet water temperatures.

Conclusions

Numerical modeling is performed for laminar upward flow with heating in a vertical tube for conditions near the critical region of water. In this region there are large variations of thermodynamic and transport properties. It is found that the buoyancy force from the density variation that is induced by heat transfer from the wall is important in both fluid flow and heat transfer. Although a small heat flux is applied and induces very small temperature gradients, the variation of properties, especially density as reflected in the gravity effect, cannot be neglected near the critical region. A small heat flux can generate recirculation in the tube core in the downstream regions along the tube. It is shown that the buoyancy force parameter Gr/Re has steep variation with temperature near the pseudocritical temperature and this steepness depends on the pressure. Because of the gravity effect on the flow characteristics, heat transfer coefficient increases with a decrease of inlet Reynolds numbers in the developing region along the vertical tube. The heat transfer coefficient also depends on the inlet fluid temperatures and pressures because of strong property variations. As the fluid temperature approaches the pseudocritical temperature, the heat transfer coefficient increases. The friction factor increases along the tube after a short distance from the entrance region. Entrance region effects persist far downstream. Buoyancy forces are very important, even at large inlet Reynolds numbers in the laminar range, indicating that mixed convection probably persists well into turbulent flow.

References

[1]Shitsmann, M. E., ''Impairment of the Heat Transmission at Supercritical Pressure,'' *Teplofizika Vysokikh Temperatur*, Vol. 1, No. 2, 1963, pp. 237–243.

[2]Yamagata, K., Nishikawa, K., Hasegawa, S., Fujii, T., and Yoshida, S., ''Forced Convective Heat Transfer to Supercritical Water Flowing in Tubes,'' *International Journal of Heat and Mass Transfer*, Vol. 15, No. 12, 1972, pp. 2575–2593.

[3]Watts, M. J., and Chou, C. T., ''Mixed Convection Heat Transfer to Supercritical Pressure Water,'' *Proceedings of the International Heat Transfer Conference*, Vol. 3, Hemisphere, New York, 1982, pp. 495–500.

[4]Kurganov, V. A., and Kaptilnyi, A. G., ''Flow Structure and Turbulent Transport of a Supercritical Pressure Fluid in a Vertical Heated Tube Under the Conditions of Mixed Convection. Experimental Data,'' *International Journal of Heat and Mass Transfer*, Vol. 36, No. 13, 1993, pp. 3383–3392.

[5]Hall, W. B., ''Heat Transfer near the Critical Point,'' *Advances in Heat Transfer*, Vol. 7, 1971, pp. 2–83.

[6]Polyakov, A. F., ''Heat Transfer Under Supercritical Pressures,'' *Advances in Heat Transfer*, Vol. 21, 1991, pp. 1–53.

[7]Koshizuka, S., Takano, N., and Oka, Y., ''Numerical Analysis of Deterioration Phenomena in Heat Transfer to Supercritical Water,'' *International Journal of Heat and Mass Transfer*, Vol. 38, No. 16, 1995, pp. 3077–3084.

[8]Zhou, N., and Krishnan, A., ''Laminar and Turbulent Heat Transfer in Flow of Supercritical CO_2,'' *Proceedings of the National Heat Transfer Conference*, Vol. 5, 1995, pp. 53–63.

[9]Koppel, L. B., and Smith, J. M., ''Laminar Flow Heat Transfer for Variable Physical Properties,'' *Journal of Heat Transfer*, Vol. 84, No. 2, 1962, pp. 157–163.

[10]Dashevsky, Y. M., and Malkovsky, V. I., ''Heat Exchange with Laminar Upflow of Supercritical Helium in a Gravitational Field,'' *Cryogenics*, Vol. 25, Nov. 1985, pp. 658, 659.

[11]Dashevsky, Y. M., Malkovsky, V. I., Miropolsky, Z. L., Sakovich, N. D., and Khasanov-agayev, L. R., ''Effect of Thermogravitation on Heat Transfer in Up or Down Pipe Flow of Supercritical Fluids,'' *Heat Transfer—Soviet Research*, Vol. 18, No. 6, 1986, pp. 109–114.

[12]Bogachev, V. A., Yeroshenko, V. M., and Yaskin, L. A., ''Heat Transfer Associated with an Ascending Flow of Supercritical Helium in a Heated Tube with Re < 2300 at the Entry,'' *Teplofizika Vysokikh Temperatur*, Vol. 21, No. 1, 1983, pp. 101–106.

[13]Vlakhov, E. S., Miropol'skii, Z. L., and Khasanov-agaev, L. R., ''Heat Transfer to a Supercritical Medium with Mixed Convection and Rising Flow in Heated Tubes,'' *Teploenergetika*, Vol. 28, No. 11, 1981, pp. 69–71.

[14]Lester, H., John, S. G., and George, S. K., *Steam Tables*, Hemisphere, New York, 1984.

[15]Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, Hemisphere, Washington, DC, 1980.

[16]WorsØe-Schmidt, P. M., and Leppert, G., ''Heat Transfer and Friction for Laminar Flow of Gas in a Circular Tube at High Heating Rate,'' *International Journal of Heat and Mass Transfer*, Vol. 8, No. 10, 1965, pp. 1281–1301.

[17]Zeldin, B., and Schmidt, F. W., ''Developing Flow with Combined Forced-Free Convection in an Isothermal Vertical Tube,'' *Journal of Heat Transfer*, Vol. 94, No. 2, 1972, pp. 211–223.

[18]Moutsoglou, A., and Kwon, Y. D., ''Laminar Mixed Convection Flow in a Vertical Tube,'' *Journal of Thermophysics and Heat Transfer*, Vol. 7, No. 2, 1993, pp. 361–368.